NONSTATIONARY HEAT TRANSFER IN A HETEROGENEOUS MEDIUM

V. A. Romanov

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We consider heat transfer in the displacement of a hot interstitial liquid from an artificially cracked region formed by a camouflet explosion.

The problem of heat transfer in a medium consisting of solid and liquid phases arises in many technical applications; in investigating the operation of a regenerative heat exchanger [1, 2], in investigating the heat-transfer process in forced filtering through an underground water-saturated stratum [3], etc. However, this problem is particularly important in the extraction of interior heat of "dry" rocks by circulation systems [4].

The basic heat-transfer element of a circulation system is the region formed by an underground camouflet explosion: a region of crushing and intense splitting. Cracks formed as a result of tensile, normal, and tangential stresses at the instant of the explosion break down the rock massif into blocks whose shapes depend on the distance from the center of the explosion. In the region immediately adjoining the cavity formed the cleavage approaches cubic, while at distances of 3-5 cavity radii the massif is split into slabs. The openings of artificial cracks vary from 5 to 0.1 mm for average slab lengths of 2 to 5 m, respectively [5].

We describe the energy transformations in such an inhomogeneous medium by using the concept of a porous medium [6], stipulating the conditions for averaging the basic thermodynamic characteristics. We assume that the scale of averaging for the thermodynamic characteristics $L \gg r_0$, where r_0 is some statistically averaged dimension of a block. The skeleton of the medium formed has a finite specific heat and $r_0 > \Delta$, where Δ is the average half-width of one filtering channel.

We consider the heat-transfer process in the displacement of a hot interstitial liquid from artificially cracked rock, using the model shown in Fig. 1. We assume that the heat flux from the solid phase is described in the energy Eq. (1) by a nonstationary heat source q_V whose strength is determined by the temperature of the filtering liquid, i.e.,

$$mG_l \frac{\partial t_l}{\partial \tau} + mG_l w_x \frac{\partial t_l}{\partial x} = \lambda_y \frac{\partial^2 t_l}{\partial y^2} + (1-m) q_v. \quad (1)$$

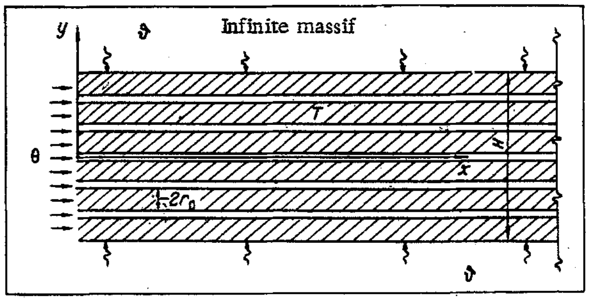

Fig. 1. Calculational model of a heterogeneous medium.

In the far zone the skeleton of the cracked rock can be thought of as consisting of flat slabs of half-width r_0 having a relative area of σ m²/m³. The corresponding heat-conduction equation for a slab is

$$\frac{\partial^2 t_p}{\partial r^2} - \frac{\partial t_p}{\partial \tau} = 0. \quad (2)$$

We assume that the temperature of the solid phase (slab) at its surface is equal to the local temperature of the liquid; i.e.,

$$t_p|_{r=1} = t_l|_{r=1}. \quad (3)$$

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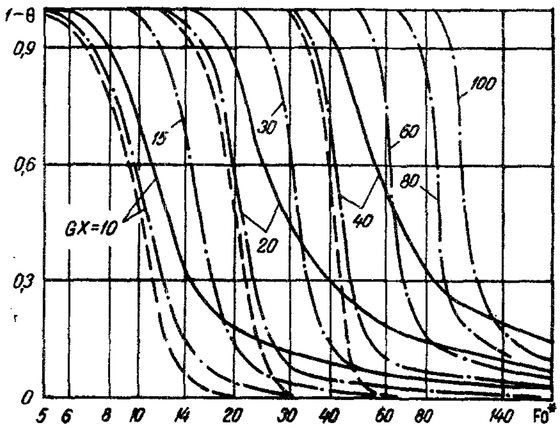

Fig. 2. Average temperature of liquid in a cracked medium. Dashed curve Eq. (16'); dash-dot curve Eq. (15) with k = 0.020; solid curve Eq. (15) with k = 0.089.

This assumption is valid for a fractured massif with narrow cracks (1 ~ = 0.1 mm) sufficiently close together [7]. In this case the time to equalize the temperature of a slab and the filtering liquid is shorter than the time to equalize the temperature over the volume of the slab itself [7, 8].

The system of equations can be written in the following dimensionless form:

$$\frac{\partial\theta}{\partial \mathrm{Fo}} + \frac{\partial\theta}{\partial X} = a\frac{\partial^2\theta}{\partial Y^2} + \mathrm{Po}(\theta), \quad 0 < Y < 1, \tag{4}$$

$$\frac{\partial\vartheta}{\partial \mathrm{Fo}_1} = \frac{\partial^2\vartheta}{\partial Y^2}, \quad 1 < Y < \infty, \tag{5}$$

$$\frac{\partial T}{\partial \mathrm{Fo}} = \frac{\partial^2 T}{\partial R^2}, \quad 0 < R < 1, \tag{6}$$

where θ, ϑ, and T are the temperatures of the liquid, the surrounding massif, and the slab, respectively. On the boundary between a slab and the massif we specify the coupling conditions

$$\left.\frac{\partial\theta}{\partial Y}\right|_{Y=1} = \lambda\left.\frac{\partial\vartheta}{\partial Y}\right|_{Y=1}, \tag{7}$$

$$\theta|_{Y=1} = \vartheta|_{Y=1}, \tag{8}$$

where $\lambda = \lambda_p/\lambda_y \approx \lambda_s/\lambda_y$.

It is known that for an arbitrary change of temperature of the surface of a slab θ(Fo) the temperature of the slab $T(\mathrm{Fo}) = T_{1*}\theta(\mathrm{Fo})$, where $T_1(\mathrm{Fo})$ is the solution of Eq. (6) for unique conditions on the surface of the slab. After finding this solution by using the Laplace–Carson transform $\overline{T}_1(s)$ {Fo $\div$ s} we determine the heat flux through the inner surface of the skeleton – the Pomerantsev member Po:

$$\overline{\mathrm{Po}}(\bar\theta) = -\sigma\frac{G_s r_0(1-m)}{G_l m}\cdot\left.\frac{\partial\overline{T}_1}{\partial R}\,\bar\theta\right|_{R=1} = -G\sqrt{s}\,\mathrm{th}\sqrt{s}\,\bar\theta(s, p). \tag{9}$$

Equations (4) and (5), transformed by the two-dimensional Laplace–Carson transform with the parameters Fo $\div$ s, $\mathrm{Fo}_1 \div$ q, and X $\div$ p for the initial conditions $\vartheta(\mathrm{Fo} = 0, X) = 0$, $\theta(\mathrm{Fo}, X = 0) = 1$, have the form

$$a\frac{d^2\bar\theta}{dY^2} + (s+p)\,\bar\theta + p - \overline{\mathrm{Po}}(\bar\theta) = 0, \quad \frac{d^2\bar\vartheta}{dY^2} - q\bar\vartheta = 0. \tag{10), (11}$$

Problem (9), (10), (11) with boundary conditions (7), (8) can be written in the form of a functional $F(\bar\theta)$:

$$F(\bar\theta) = \int_0^1 [a(\bar\theta_y')^2 + (s + p + G\sqrt{s}\,\mathrm{th}\sqrt{s})\,\bar\theta^2 - 2p\bar\theta]\,dY + g\sqrt{q}\,\bar\theta^2|_{Y=1}. \tag{12}$$

This relation is established after finding the variation of the functional with respect to $\bar\theta$; the integrand in this case will satisfy Euler's equation.

It is known [8] that the variational method can be used to find both approximate and exact solutions of the problem. In our case the method of undetermined coefficients can be used to find an exact solution for the temperature of the liquid $\tilde\theta$(Fo, X) averaged over the cross section. According to the Ritz method

$$\frac{\partial F}{\partial\bar\theta} = 0; \quad \bar\theta(s, p) = \frac{p}{p + s + G\sqrt{s}\,\mathrm{th}\sqrt{s} + g\sqrt{q}}. \tag{13}$$

The intermediate inverse transform $\tilde\theta^*(s, X)$ is found from the Bateman tables [9]:

$$\tilde\theta^*(s, X) = \exp\left[-sX - \frac{gH}{2r_0G}GX\sqrt{s}\right]\exp[-GX\sqrt{s}\,\mathrm{th}\sqrt{s}]. \tag{14}$$

Using the inverse transform of the function $\exp(-b\sqrt{s}\,\mathrm{th}\sqrt{s})$ found in [11] and the convolution theorem for transforms, we obtain after differentiating under the integral sign

$$\bar{\theta}(\mathrm{Fo}^*, GX) = \frac{\varkappa}{\sqrt{\pi}} \int_0^1 U_0(GX, u\cdot\mathrm{Fo}^*) \frac{\exp\left(-\frac{\varkappa^2}{1-u}\right)}{(1-u)\sqrt{1-u}} du, \tag{15}$$

where

$$\varkappa = k\cdot GX/\sqrt{\mathrm{Fo}^*};$$

$$U_0(GX, u\cdot\mathrm{Fo}^*) = \frac{1}{2} + \frac{2}{\pi} \int_0^\infty \exp[-GX\cdot f_1(x)] \sin\varphi_1(x) \frac{dx}{x},$$

$$f_1(x) = \frac{x}{2} \frac{\mathrm{sh}\, x - \sin x}{\mathrm{ch}\, x + \cos x}; \tag{16}$$

$$\varphi_1(x) = u\frac{x^2\mathrm{Fo}^*}{2} - GX\frac{x}{2}\frac{\mathrm{sh}\, x + \sin x}{\mathrm{ch}\, x + \cos x}.$$

The notation Fo*, GX, and k represent dimensionless quantities in the heat-transfer process. For negative values of Fo* the function vanishes; i.e., the temperature of the liquid at coordinate X is equal to the initial temperature of the rock massif t_p^i. The dimensionless combination of formulas (15) contains the thermophysical characteristics of both phases in accordance with the physical meaning of the heat-exchange process.

For a slab with thermally insulated lateral surfaces the temperature as a function of X is given by Eq. (16) with the parameter u set equal to unity. This follows at once from the functional (12) in which the last term describing the external effect must be set equal to zero and then

$$\bar{\theta}(s, p) = p(p + s + G\sqrt{s}\,\mathrm{th}\sqrt{s})^{-1} \div U_0(GX, \mathrm{Fo}^*). \tag{16'}$$

The results of a calculation on an M-222 computer using Eqs. (15) and (16') are shown in Fig. 2. As the thickness of the slab increases the parameter k decreases, the temperature graph of Eq. (15) approaches the graph of (16'), and thus the inflow of heat from the surrounding massif is decreased. A calculational analysis showed that for $k < 0.01$ the graphs do not differ by more than 10%, and, consequently, in certain cases the temperature can be adequately approximated by Eq. (16').

The graphs of Eq. (16') show that the general heat-transfer laws such as the constant velocity of the midpoint of a heat wave are satisfied for filtration through a porous medium [10]. The combination GX characterizes the inertia of a cracked medium; a medium with a small value of GX responds to a disturbance more quickly than one having a large value of GX. In the latter case the rate of decrease of temperature with respect to X increases, while from a certain value of Fo*, $\bar{\theta}$ varies almost linearly with GX.

Within the framework of the mathematical model assumed it is possible to calculate heat transfer in a heterogeneous medium and to estimate the parameters of a nonstationary underground heat boiler.

NOTATION

m	is the porosity fraction;
τ	is the time;
λ_y	is the effective thermal conductivity of slab along y axis;
H	is the height of slab;
t_p^i	is the initial temperature of solid phase;
t_0^{in}	is the temperature at entrance to slab;
a_l, a_s	are the thermal diffusivities of the liquid and solid phases, respectively;
G_i	is the volumetric heat capacity of i-th phase;
σ	is the relative area of cracks, m^2/m^3;
$2r_0$	is the height of slab;
w_x	is the rate of filtration of liquid along x axis.

Dimensionless Parameters

$\theta = (t_l - t_p^i)/(t_0^{in} - t_p^i)$	is the temperature of the liquid phase;
$T = (t_s - t_p^i)/(t_0^{in} - t_p^i)$	is the temperature of the solid phase;

$R = r/r_0$;
$Y = y/0.5\ H$;
$Fo = a_S\tau/r_0^2$;
$Fo_1 = a_S\tau/0.25\ H^2$;
$X = a_S x/w_X r_0^2$;
$Fo^* = Fo - X$;
$g = a \cdot \lambda$;
$a = \lambda_y r_0^2/(G_l m \cdot 0.25\ H^2)$;
$\lambda = \lambda_S/\lambda_y$;
$\kappa = gHX/[2r_0(Fo^*)^{1/2}]$;
$G = G_S\sigma r_0(1 - m)/(G_l m)$;
$\bar{\theta}'_y = d\bar{\theta}/dy$;
$T_{1*}\theta = \int_0^{Fo} T_1(\varepsilon)\theta(Fo - \varepsilon)d\varepsilon$ is the convolution of the functions T_1 and θ;

Indices

l pertains to the liquid phase;

T pertains to the solid phase.

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